

## DEPTH BY THE RULE

G.McC. Haworth<sup>1</sup>

UK

### ABSTRACT

This note corrects a previous treatment of algorithms for the metric DTR, Depth by the Rule.

Haworth (2000) addressed optimal winning strategies for chess endgames, given the constraint of the 50-move rule. It proposed the revival of the Depth to Zeroing-move<sup>2</sup> metric DTZ, to be used with the familiar DTC, Depth to Conversion, and DTM(ate) metrics. It also introduced the new DTR metric, Depth by the Rule.

A depth under the DTX rule is denoted by  $dx$  here:  $dr$  is the least  $k$  under which a position can be won with best play under a  $k$ -move rule.  $dm \geq dr \geq dz$  while  $dr = dz$  if  $dz \geq \max\{dz\}$  for all subsequent endgames. Mednis (1996) highlights the Shamkovich-Wirthensohn (Biel, 1980) KQPKQ position wKe7Qf6Ph4/bKc7Qg3+w. This could have led to a 50-move draw claim with the sides minimaxing  $dz$  but appears to avoid the claim if  $dc$  is minimaxed. DTR endgame tables (EGTs) determine whether such 50-move draw claims are avoidable.

In an ill-advised attempt to create an easily-understood DTR algorithm AL1,  $dr$  was initialised to  $dz$  across the endgame rather than selectively, and the following incorrect recurrence relations were adopted:

- $dr = \max [dz, \min (dr \text{ of won successors})]$  for side-to-move,  $stm$ , wins
- $dr = \max [dz, \max (dr \text{ of lost successors})]$  for  $stm$  losses

The relations assume wrongly that any move sequence minimaxing  $dr$  starts with a sequence minimaxing  $dz$ . KRRKB position wKa1Ra2b1/bKd1Bc1+w is a counterexample;  $dz = 1$  ply (1. Rxc1+) while  $dr = 3$  plies (1. Rab2 Ke1 2. Rxc1#) and  $\min(dr \text{ of won successors}) = 2$  plies.

In fact, the standard retrograde-analysis algorithm which has already produced EGTs for the DTC, DTZ and DTM metrics (Thompson, 1986) can be used to produce EGTs to the DTR metric. The most familiar DTC version sets  $dc$  in the first possible phase, and the same holds for the DTZ version and  $dz$ . For the DTM metric,  $dm$  may be set in the first possible phase and potentially reduced later, or more efficiently (Wu and Beal, 2001), all positions with  $DTM = dm$  can be given their depth correctly in phase  $dm$ .

The DTR computation requires the parameter  $dzt$  (plies), “Depth to Zeroing-move while minimaxing DTR”, rather than  $dz$ <sup>3</sup>. Position P[ $dr, dzt$ ] potentially backs up to P[ $\max(dr, dzt+1), dzt+1$ ]. The proof that the retrograde algorithm is correct for DTM and DTR, as well as for DTC and DTZ, is arguably non-trivial and ultimately uses the fact that it is a correctly-initialised, discrete, finite process where any error in the results must be preceded by an earlier error, an impossibility.

In conclusion, algorithm AL1 (Haworth, 2000) should be discounted as incorrect and AL2-3 are perhaps needlessly conservative and inefficient as they are constrained to identify won positions in increasing  $dr$  order.

### REFERENCES

- Haworth, G.McC. (2000). Strategies for Constrained Optimisation. *ICGA Journal*, Vol. 23, No. 1, pp. 9-20.
- Mednis, E. (1996). *Advanced Endgame Strategies*, esp. 94-96. Chess Enterprises. ISBN 0-9454-7059-2.
- Thompson, K. (1986). Retrograde Analysis of Certain Endgames. *ICCA Journal*, Vol. 9, No. 3, pp. 131-139.
- Wu, R. and Beal, D. (2001). Solving Chinese chess endgames by database construction. *Information Sciences*, Vol. 135, pp. 207-228.

<sup>1</sup> 33, Alexandra Rd., Reading, Berkshire, RG1 5PG, UK. email: guy.haworth@icl.com

<sup>2</sup> Pawn-pushes and/or captures which set the move-count to zero.

<sup>3</sup> Note that  $dzt$  may be  $>$  (as above),  $=$  or  $<$   $dz$  as for wKa1Qf2Ra3/bKg1+b: Black may capture voluntarily.