

DEPTH BY THE RULE*G.McC. Haworth¹*

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ABSTRACT

This note corrects a previous treatment of algorithms for the metric DTR, Depth by the Rule.

Haworth (2000) addressed optimal winning strategies for chess endgames, given the constraint of the 50-move rule. It proposed the revival of the Depth to Zeroing-move² metric DTZ, to be used with the familiar DTC, Depth to Conversion, and DTM(ate) metrics. It also introduced the new DTR metric, Depth by the Rule.

A depth under the DTX rule is denoted by dx here: dr is the least k under which a position can be won with best play under a k -move rule. $dm \geq dr \geq dz$ while $dr = dz$ if $dz \geq \max\{dz\}$ for all subsequent endgames. Mednis (1996) highlights the Shamkovich-Wirthensohn (Biel, 1980) KQPKQ position $wKe7Qf6Ph4/bKc7Qg3+w$. This could have led to a 50-move draw claim with the sides minimaxing dz but appears to avoid the claim if dc is minimaxed. DTR endgame tables (EGTs) determine whether such 50-move draw claims are avoidable.

In an ill-advised attempt to create an easily-understood DTR algorithm AL1, dr was initialised to dz across the endgame rather than selectively, and the following incorrect recurrence relations were adopted:

- $dr = \max [dz, \min (dr \text{ of won successors})]$ for side-to-move, stm , wins
- $dr = \max [dz, \max (dr \text{ of lost successors})]$ for stm losses

The relations assume wrongly that any move sequence minimaxing dr starts with a sequence minimaxing dz . KRRKB position $wKa1Ra2b1/bKd1Bc1+w$ is a counterexample; $dz = 1$ ply (1. Rxc1+) while $dr = 3$ plies (1. Rab2 Ke1 2. Rxc1#) and $\min(dr \text{ of won successors}) = 2$ plies.

In fact, the standard retrograde-analysis algorithm which has already produced EGTs for the DTC, DTZ and DTM metrics (Thompson, 1986) can be used to produce EGTs to the DTR metric. The most familiar DTC version sets dc in the first possible phase, and the same holds for the DTZ version and dz . For the DTM metric, dm may be set in the first possible phase and potentially reduced later, or more efficiently (Wu and Beal, 2001), all positions with $DTM = dm$ can be given their depth correctly in phase dm .

The DTR computation requires the parameter dzt (plies), “Depth to Zeroing-move while minimaxing DTR”, rather than dz^3 . Position $P[dr, dzt]$ potentially backs up to $P[\max(dr, dzt+1), dzt+1]$. The proof that the retrograde algorithm is correct for DTM and DTR, as well as for DTC and DTZ, is arguably non-trivial and ultimately uses the fact that it is a correctly-initialised, discrete, finite process where any error in the results must be preceded by an earlier error, an impossibility.

In conclusion, algorithm AL1 (Haworth, 2000) should be discounted as incorrect and AL2-3 are perhaps needlessly conservative and inefficient as they are constrained to identify won positions in increasing dr order.

REFERENCES

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² Pawn-pushes and/or captures which set the move-count to zero.

³ Note that dzt may be $>$ (as above), $=$ or $<$ dz as for $wKa1Qf2Ra3/bKg1+b$: Black may capture voluntarily.